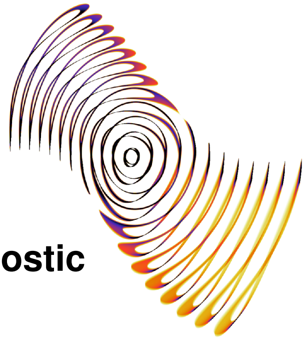

A synthetic synchrotron diagnostic for runaways in tokamaks

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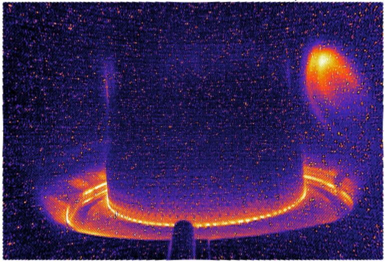
Outline

1. Theory of our synthetic diagnostic
2. Geometric effects
3. Image sensitivity to RE parameters
4. Modelling C-Mod discharge



Synthetic synchrotron diagnostic theory

Total power per pixel, per frequency interval $\mathrm{d}\omega$:

$$\frac{\mathrm{d}I_{ij}}{\mathrm{d}\omega}(\boldsymbol{x}_0,\omega) = \int \mathrm{d}\boldsymbol{x}\mathrm{d}\boldsymbol{p} \int_A \mathrm{d}A \int_{\boldsymbol{N}_{ij}} \mathrm{d}\boldsymbol{n} \times$$
$$\times \frac{\hat{\boldsymbol{n}}\cdot\boldsymbol{n}}{r^2} f(\boldsymbol{x},\boldsymbol{p})\delta\left(\frac{\boldsymbol{r}}{r} - \boldsymbol{n}\right) \frac{\mathrm{d}^2 P(\boldsymbol{x},\boldsymbol{p},\boldsymbol{x}_0,\omega)}{\mathrm{d}\omega\mathrm{d}\Omega}$$

Detector parameters

A = Detector surface,
$\boldsymbol{n}$ = Line-of-sight
$\hat{\boldsymbol{n}}$ = Viewing direction
$\boldsymbol{x}_0$ = Detector position

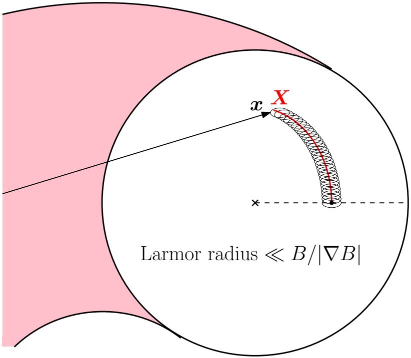
$\mathrm{d}^2 P/\mathrm{d}\omega\mathrm{d}\Omega$ = Angular and spectral distribution of synchrotron radiation,

$f(\boldsymbol{x},\boldsymbol{p})$ = Distribution of runaways,

Particle parameters

$r = |\boldsymbol{x} - \boldsymbol{x}_0|$ = Distance between camera and particle.

Synthetic synchrotron diagnostic theory

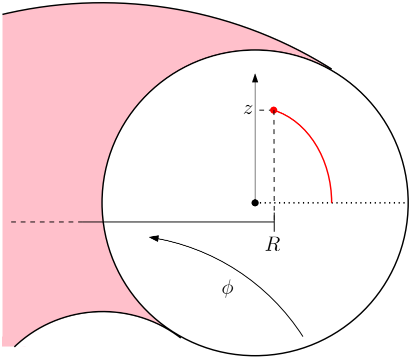


Three transformations

1. Guiding-center approx.,

$$\mathrm{d}\boldsymbol{x}\mathrm{d}\boldsymbol{p} \approx \mathrm{d}\boldsymbol{X}\mathrm{d}p_{\|}\mathrm{d}p_{\perp}\mathrm{d}\zeta$$

Synthetic synchrotron diagnostic theory



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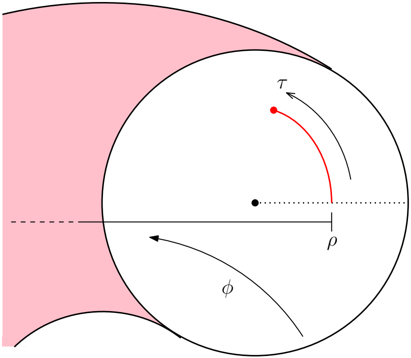
$$\mathrm{d}\boldsymbol{X} = R\,\mathrm{d}R\mathrm{d}z\mathrm{d}\phi$$

Synthetic synchrotron diagnostic theory



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3. Trajectory coordinates $(R, z) \to (\rho, \tau)$,
   - ρ: Major radius of particle in the midplane, at **beginning** of orbit
   - τ: Orbit time (a poloidal parameter)

Synthetic synchrotron diagnostic theory

Distribution function independent of:

- Toroidal angle ϕ – Tokamak axisymmetry
- Gyrophase ζ – Gyrotropy
- Orbit time τ – Liouville's theorem

Guiding-center distribution specified along the line $\tau = \phi = 0$ (outer midplane).

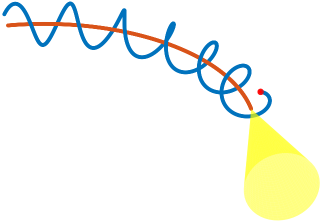
$$\frac{\mathrm{d}I_{ij}}{\mathrm{d}\omega} = \int_A \mathrm{d}A \int_{\boldsymbol{N}_{ij}} \mathrm{d}\boldsymbol{n} \int \mathrm{d}\rho\mathrm{d}\tau\mathrm{d}\phi\mathrm{d}p_{\|}\mathrm{d}p_{\perp} \times p_{\perp} JR \times \\ \times \frac{\hat{\boldsymbol{n}}\cdot\boldsymbol{n}}{r^2} f_{\mathrm{gc}}(\rho, p_{\|}, p_{\perp})\delta\left(\frac{\boldsymbol{r}}{r} - \boldsymbol{n}\right)\left\langle \frac{\mathrm{d}^2 P(\rho, p_{\|}, p_{\perp}, \boldsymbol{x}_0, \omega)}{\mathrm{d}\omega\mathrm{d}\Omega}\right\rangle$$

Synchrotron radiation

Angular and spectral distribution of synchrotron radiation:

$$\frac{d^2P}{d\omega d\Omega} = \frac{3e^2\beta^2\gamma^6\omega_B}{32\pi^3\epsilon_0 c}\left(\frac{\omega}{\omega_c}\right)^2\left(\frac{1-\beta\cos\psi}{\beta\cos\psi}\right)^2 \times$$
$$\times\left[K^2_{2/3}(\xi) + \frac{(\beta/2)\cos\psi\sin^2\psi}{1-\beta\cos\psi}K^2_{1/3}(\xi)\right]$$

Result of gyro-average:



SOFT – Sychrotron-detecting Orbit Following Toolkit

- Computes $\mathrm{d}I_{ij}/\mathrm{d}\omega$, and outputs synchrotron images and spectra

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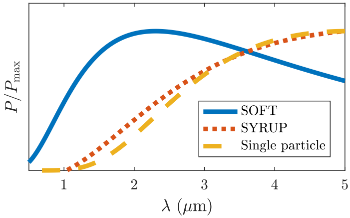
- Computes $\mathrm{d}I_{ij}/\mathrm{d}\omega$, and outputs synchrotron images and spectra
- Solves the guiding-center equations of motion using RKF45 in numeric magnetic geometry
- Weighted with a given (numeric) runaway distribution function
- Full distribution runs in 5-10 hours on 4-core Xeon-based desktop, with sufficient resolution

Comparison with SYRUP [1]

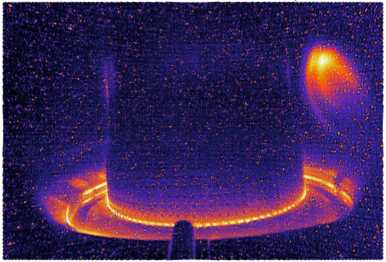


- Geometric effects (SOFT) show significant difference in spectrum.
- Runaway distribution specified explicitly in outer-midplane (LF-side).
- Contributions mostly from HF-side.

[1] A. Stahl, et. al. PoP 20, 093302 (2013).

Parameter scans

C-Mod 1140403026, t ~ 0.742 s



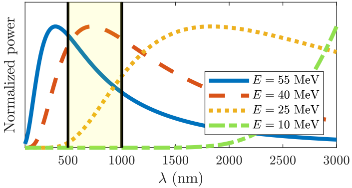
Magnetic geometry:
Alcator C-Mod, 3-8 T

- Radiation in the visible range
- Camera located 21 cm below midplane

Varied parameters:

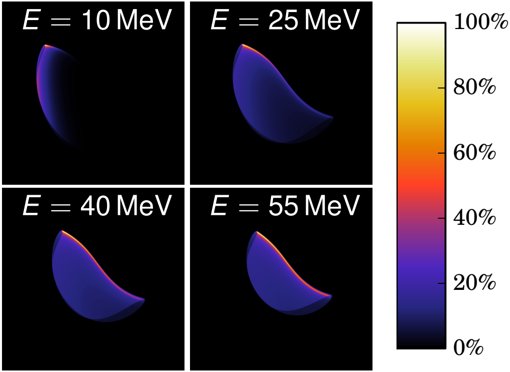
- Energy E
- Pitch angle θ_{p}
- Initial radius

Parameter scans – Energy



Other parameters:

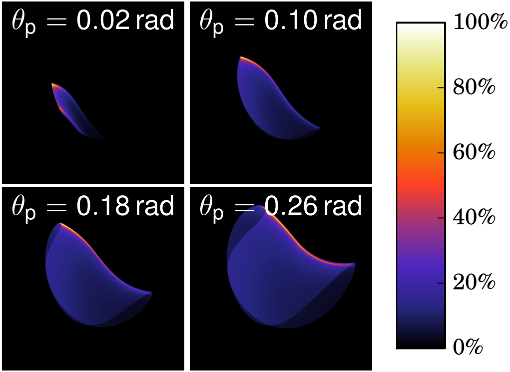
Beam radius	16 cm
Pitch angle	0.15 rad
Spectral range	500-1000 nm
Magnetic field	3-8 T
Camera elevation	-21 cm



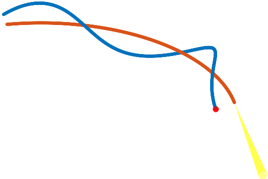
Parameter scans – Pitch angle

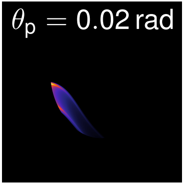
Other parameters:

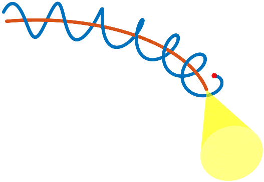
Beam radius	16 cm
Energy	30 MeV
Spectral range	500-1000 nm
Magnetic field	3-8 T
Camera elevation	−21 cm

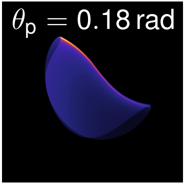


Small pitch angle = small GC cone	**Large pitch angle = large GC cone**
$\Longrightarrow$ small chance of reaching detector	$\Longrightarrow$ greater chance of reaching detector









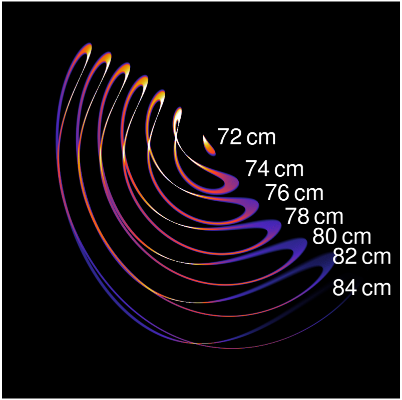
Parameter scans – Launch radius

Other parameters:

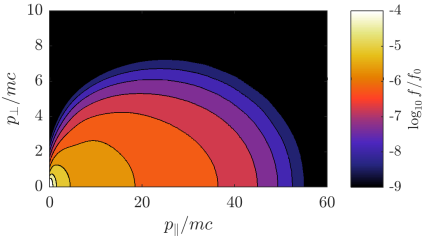
Beam radius	16 cm
Energy	30 MeV
Pitch angle	0.15 rad
Spectral range	500-1000 nm
Magnetic field	3-8 T
Camera elevation	−21 cm

NOTE: Magnetic axis at $\boldsymbol{R = 68}$ **cm**.

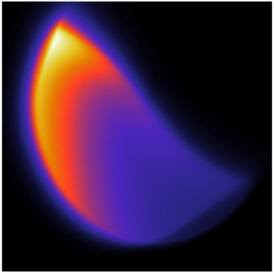
Particles at $R \lesssim 72$ cm are invisible in this configuration.



Distribution function



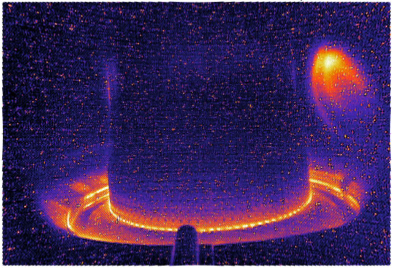
→

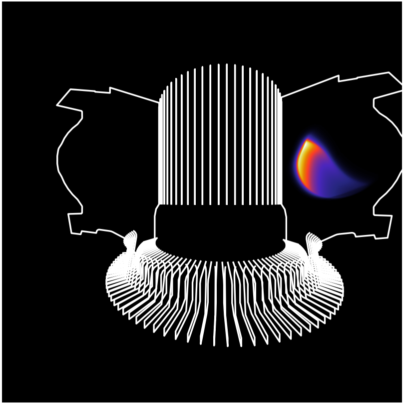


- Simulated with CODE [2, 3]
- Parameters given on-axis

[2] M. Landreman, et. al. CPC 185, 847 (2014).
[3] A. Stahl, et. al. NF 56, 112009 (2016).

Distribution function

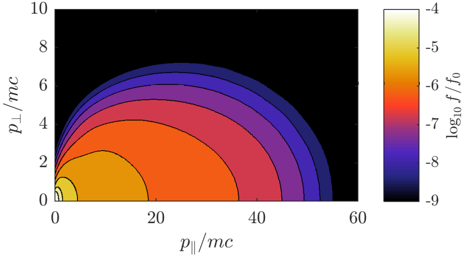




What do we actually see?

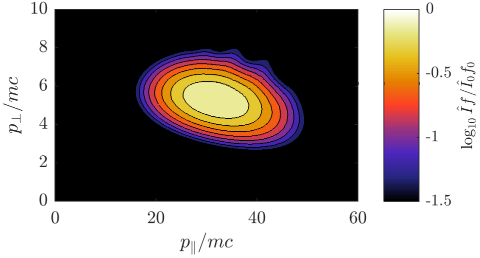
$$f(p_\parallel, p_\perp)$$

(Distribution function)



$$\text{“}\hat{I} \times f(p_\parallel, p_\perp)\text{”}$$

(Emitted radiation)



Conclusions

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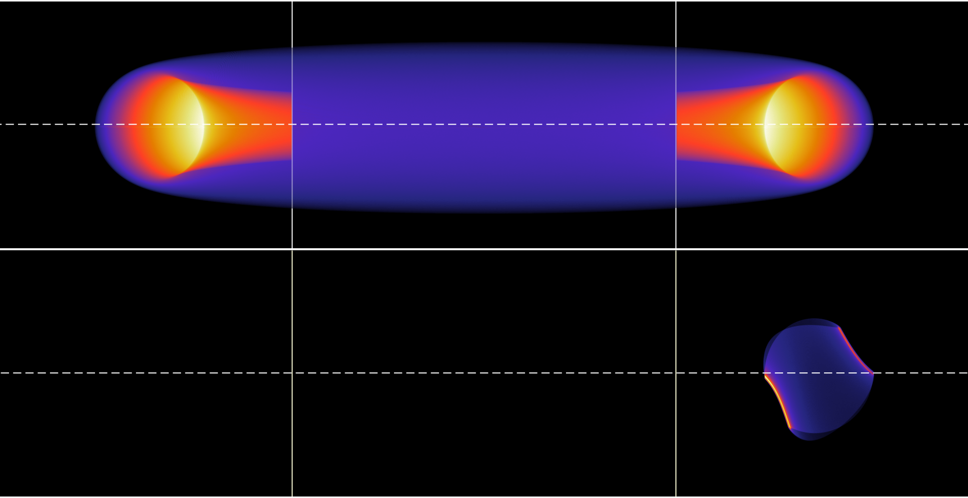
- Pitch angle varies along orbit $\implies$ crucial to be clear about how the runaway distribution is specified.
- Detector placement strongly influences the observed synchrotron radiation.
- Sensitivity due to runaway properties helps inferring runaway distribution from image.

EXTRA SLIDES



Parameter scans – Camera vertical position

Other parameters:

Beam radius	16 cm
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Pitch angle	0.15 rad
Spectral range	500-1000 nm
Magnetic field	3-8 T