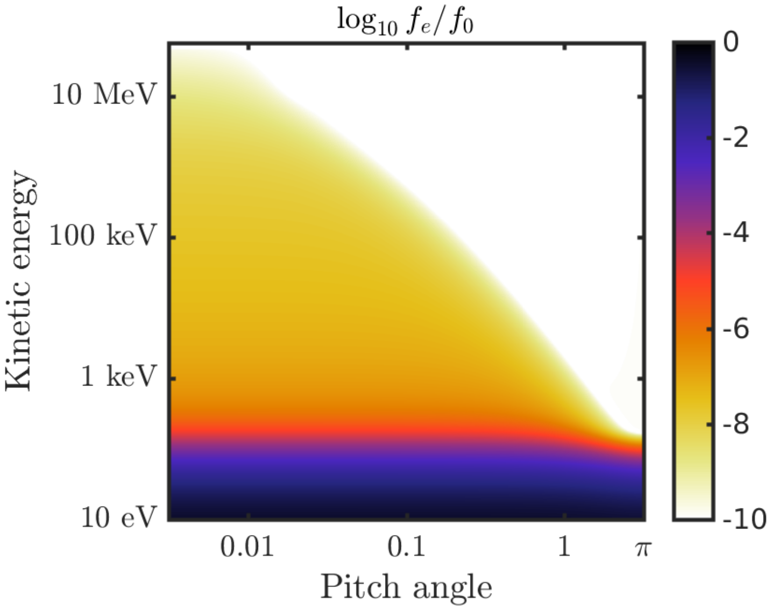
$\log_{10} f_e/f_0$
10 MeV
100 keV
1 keV
10 eV
Kinetic energy
0.01
0.1
1
π
Pitch angle
0
-2
-4
-6
-8
-10

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Improved fluid models for runaway generation and decay

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The runaway fluid

$$j_{\mathrm{RE}} = -en_{\mathrm{RE}}\langle v_{\|}\rangle$$
$$\langle v_{\|}\rangle = \frac{1}{n_{\mathrm{RE}}}\int \mathrm{d}\mathbf{p}\, v_{\|} f_{\mathrm{RE}}.$$

The runaway current evolution is given by

$$\frac{\mathrm{d}j_{\mathrm{RE}}}{\mathrm{d}t} = -ecn_{\mathrm{RE}}\Big[\Gamma(E,\,t)u(E,\,t) + \frac{\mathrm{d}}{\mathrm{d}t}u(E,\,t)\Big],$$
$$\Gamma(E,\,t) \equiv \frac{1}{n_{\mathrm{RE}}}\frac{\mathrm{d}n_{\mathrm{RE}}}{\mathrm{d}t},$$
$$u(E,\,t) \equiv \langle v_{\|}\rangle/c.$$

The runaway fluid

Kinetic simulations unnecessary whenever

$$\Gamma(E,\, t) = \Gamma(E(t))$$
$$u(E,\, t) = u(E(t)),$$

i.e. when the system is in momentaneous steady-state.

$$\frac{1}{j_{\rm RE}}\frac{{\rm d}j_{\rm RE}}{{\rm d}t} = \Gamma(E) + \frac{1}{u(E)}\frac{\partial u}{\partial E}\frac{{\rm d}E}{{\rm d}t}$$

The runaway fluid

Requires slowly varying parameters

$\rightarrow$ May be the case during the current quench

Collision time of relativistic electron : $\tau_c = \dfrac{4\pi\varepsilon_0^2 m_e^2 c^3}{\ln\Lambda n_e e^4} = \dfrac{m_e c}{eE_c}$

Decay time of current : $\tau_\text{decay} \sim \dfrac{j_\text{RE}}{\partial j_\text{RE}/\partial t} \approx \dfrac{j_\text{RE}\hat{L}}{E_c}$

$$\frac{\tau_\text{decay}}{\tau_c} \sim \frac{e\mu_0 I_\text{RE}}{2\pi m_e c} \approx \frac{I_\text{RE}}{8.5\,\text{kA}} \gg 1$$

Example: Let's compare time-dependent vs steady state growth rates

$$E(t) = \left[2 - 0.8\frac{t}{t_{\Delta E}}\right] E_c$$

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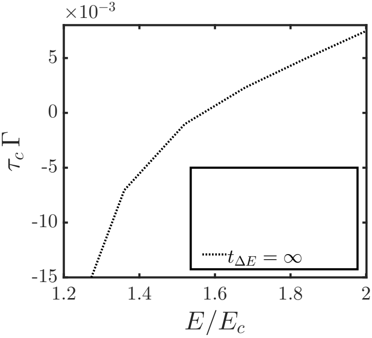
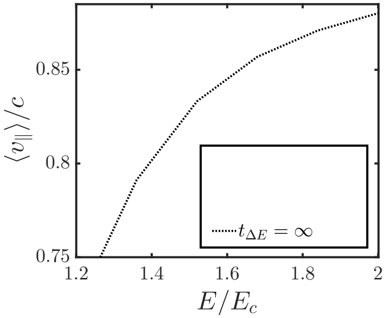
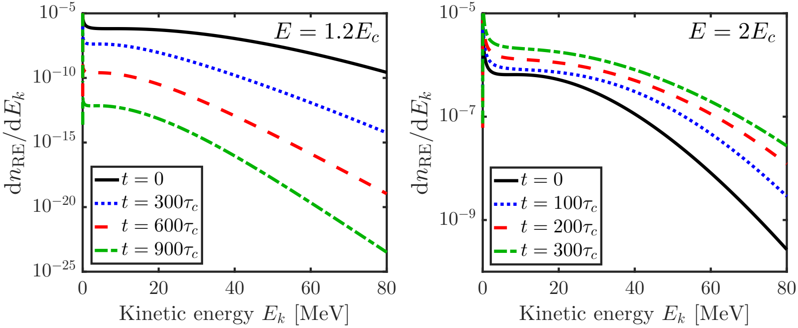
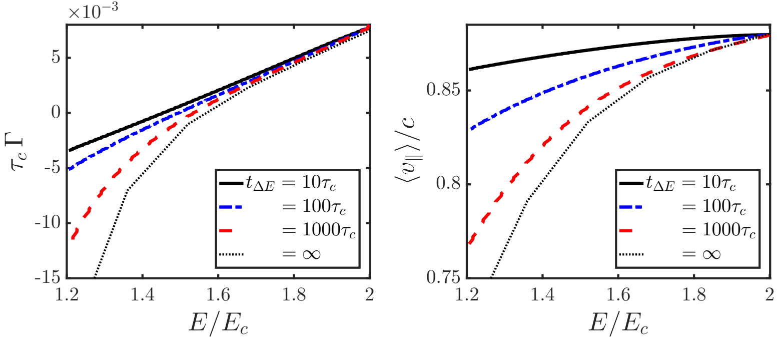




Self-similar evolution occurs both for growth and decay:



$$E(t) = \left[2 - 0.8\frac{t}{t_{\Delta E}}\right] E_c$$



When quasi-steady state is valid, the mission of kinetic theory is *only* to determine $\Gamma(E, ...)$
(and to a lesser extent $\langle v_{||} \rangle$)

How do we determine Γ as accurately as possible?

Avalanche generation

To describe knock-on collisions we add a (simplified) Boltzmann operator:

$$\frac{\mathrm{d}f_e}{\mathrm{d}t} = C_{\mathrm{FP}}\{f_e\} + C_{\mathrm{boltz}}\{f_e\},$$

$$C_{\mathrm{boltz}}\{f_a,\, f_b\}(\mathbf{p}) = \int \mathrm{d}\mathbf{p}_1 \int \mathrm{d}\mathbf{p}_2\, \frac{\partial\sigma_{ab}}{\partial\mathbf{p}} v_{\mathrm{rel}} f_a(\mathbf{p}_1) f_b(\mathbf{p}_2) - f_a(\mathbf{p}) \int \mathrm{d}\mathbf{p}'\, v_{\mathrm{rel}}\sigma_{ab}(\mathbf{p},\, \mathbf{p}') f_b(\mathbf{p}')$$

Generally we can linearize ($n_{\mathrm{RE}} \ll n_e$)

$$C_{\mathrm{boltz}}\{f_e,\, f_e\} \approx \underbrace{C_{\mathrm{boltz}}\{f_e,\, f_{e0}\}}_{\text{test-particle}} + \underbrace{C_{\mathrm{boltz}}\{f_{e0},\, f_e\}}_{\text{field-particle}}.$$

Avalanche generation

The two most established knock-on models today:

$$C_{\text{knock-on}} = C_{\text{boltz}}\{n_e\delta(\mathbf{p}),\ f_e\} \quad \text{(only field-particle term)}$$

Rosenbluth-Putvinski: $$f_e(\mathbf{p}) = n_{\text{RE}} \lim_{p_0\to\infty} \frac{1}{p^2}\delta(p-p_0)\delta(\cos\theta - 1)$$

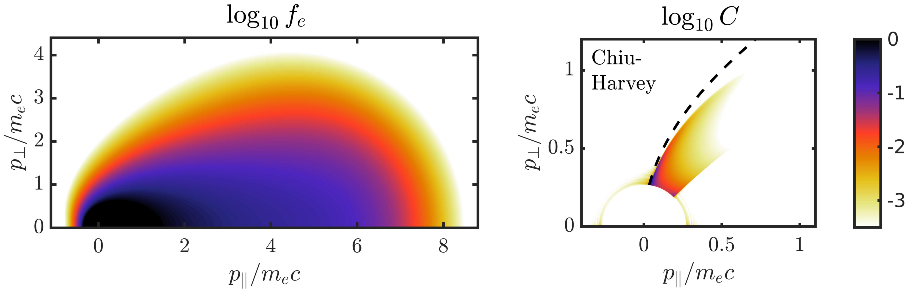
Chiu-Harvey: $$f_e(\mathbf{p}) = F(p)\delta(\cos\theta - 1)$$

$$\left(F(p) = \int_{-1}^{1} f_e(\mathbf{p})\,\mathrm{d}(\cos\theta)\right)$$

[Rosenbluth, Putvinski NF 1997; Chiu, Rosenbluth, Harvey NF 1998]

Avalanche generation

So how do these operators behave?



Avalanche generation

Both models have limitations:

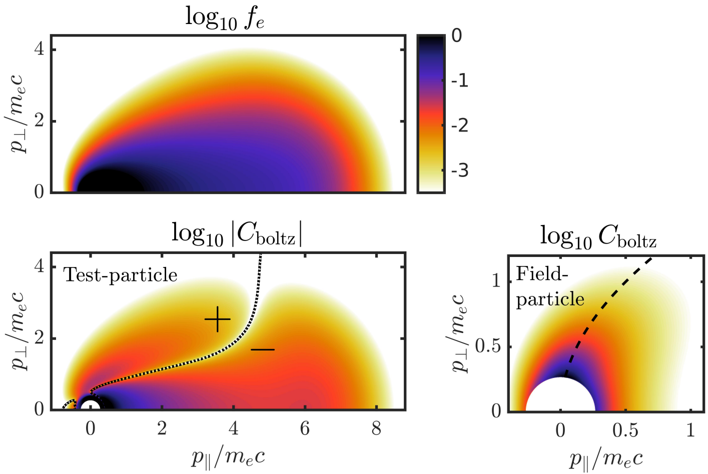
- Double counting collisions
- Non-conservation of momentum and energy
  - – Rosenbluth-Putvinski even creates infinite energy and momentum!
- Chiu-Harvey model ignores pitch-angle distribution
- Arbitrary cut-off affecting solutions

Avalanche generation

We solved this, by

- Accounting for full $f_e(\mathbf{p})$
- Including the test-particle term [restores conservation laws]
- Modify $\ln\Lambda$ in Fokker-Planck operator [avoids double counting]

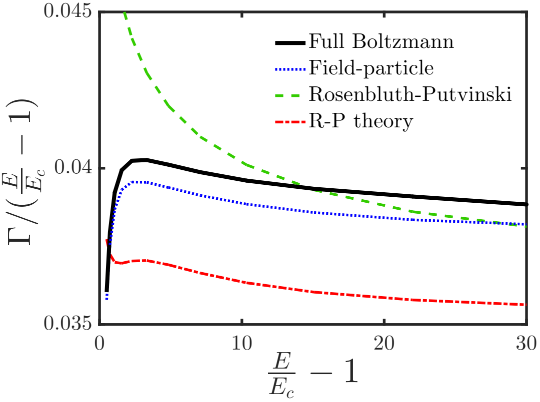
Avalanche generation



Avalanche generation

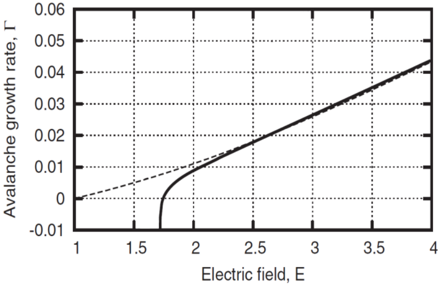
We can now revisit a classical calculation [R-P, NF 1998]:
The steady state avalanche growth rate

$$\Gamma = \frac{1}{n_{\mathrm{RE}}}\frac{\mathrm{d}n_{\mathrm{RE}}}{\mathrm{d}t}$$



Avalanche generation in a near-threshold electric field

An interesting situation occurs when $E \sim E_c$, as radiation losses become important.



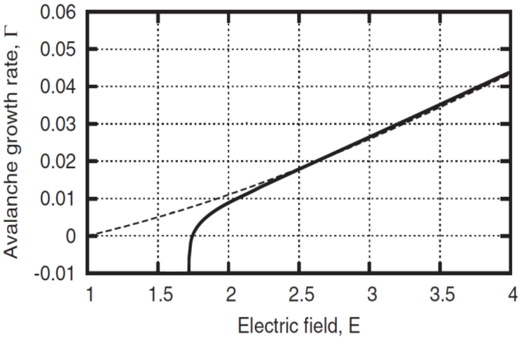
[P. Aleynikov and B. N. Breizman, PRL **114**, 155001 (2015)]

Near-threshold electric field

Approximate Γ calculated from the avalanche cross-section

$$\Gamma(E) \approx v \int_{\gamma_{min}}^{\gamma_{max}} \frac{\partial \sigma}{\partial \gamma} \mathrm{d}\gamma.$$

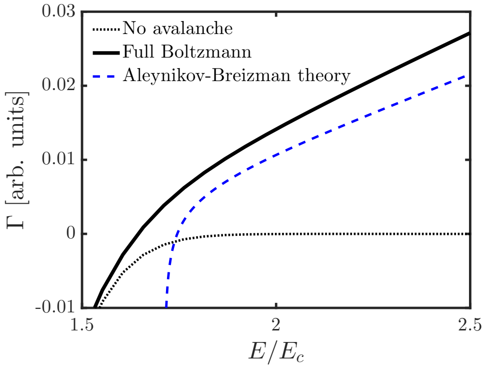
Negative growth for small *E*: Reverse knock-ons predicted!



[P. Aleynikov and B. N. Breizman, PRL **114**, 155001 (2015)]

Near-threshold electric field

- Significant reverse knock-on however ***not*** observed in kinetic simulations
- Runaway decay is described mainly by Fokker-Planck dynamics when $\Gamma \lesssim 0$.



Summary

- **Runaway fluids**
  - Strictly valid when background variations slow (for example current quench)
  - Accuracy then only limited by the kinetics used to find $\Gamma(E, ...)$
  - Runaway dissipation can be described in the fluid picture
- **Avalanche runaway modelling**
  - Conservative knock-on operator from Boltzmann
  - Formally eliminates double counting collisions, and describes reverse knock-on

Runaway kinetic theory is here to stay.